NUMERICAL SIMULATION OF A VISCOELASTIC FLOW THROUGH A CONCENTRIC ANNULAR – INFLUENCE OF THE DEBORAH NUMBER

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Abstract. The success of oil well drilling process depends on the correct prediction of the velocity and stresses fields inside the gap between the drill string and the rock formation. Using CFD is possible to predict the behavior of the drilling fluid flow along the annular space, from the bottom to the top of the well. Commonly the drilling fluid is modeled as a viscoplastic fluid. However, in this paper an alternative non-linear viscoelastic model is employed. The Phan-Thien-Tanner constitutive equation (PTT) is used to model the drilling fluid flow along the annular space. Thus, this work investigates the influence of the Deborah number on the laminar flow pattern through the numerical solution of the equations formed by the coupled velocity-pressure-stress fields. The results are analyzed and validated against the analytical solution for the fully developed annular pipe flow. The main difficulty in simulating viscoelastic flows is the intense numerical instability encountered when the elastic parameters of the fluid are increased. To avoid these instabilities the Both Sides Diffusion scheme - BSD is implemented. It is showed that the BSD scheme reduces in a very significant way the number of iterations to reach the steady state, making the convergence process more stable and faster. The relation between the Deborah number and the flow developing length is investigated, along with the influence of high values of Deborah number on the friction factor, stress and velocity fields.

Keywords: annular flow, viscoelastic flow, PTT model, BSD scheme, CFD

1. INTRODUCTION

The non-Newtonian fluid flows are present in many industrial sectors. For instance, the oil, food, chemistry and melt polymers industries usually work with non-newtonian liquid solutions in their processes.

There are many rheological models which intend to represent the non-newtonian fluid flow behavior. These models are classified in groups, according to the characteristics of the fluids. Most non-Newtonian fluid flows are modeled by different viscosity functions for the generalized Newtonian fluid model. Bingham, Power Law and Herschel-Bulkley are some viscosity functions commonly employed. Despite the satisfactory results obtained with the generalized Newtonian fluid model in many drilling fluid flow situations, in the present work, a more complex model, which can better detail the drilling fluid dynamics, will be employed.

Rheological models able to represent the behavior of fluids exhibiting both viscous and elastic characteristics are named viscoelastic.

The PTT viscoelastic model (Phan-Thien et al., 1977) is based on the polymeric net theory and considers the polymeric chains non-affine movements. Besides that, this model considers the elastic and viscous effects in the flow. PTT model was tested by many researchers, like Tanner (2002), and they classified it as one of the models which best approach the experimental data, been recommended for simulation of shear flows, like the axial annular flow.

Some decades of experience with numerical simulation of viscoelastic fluid flow showed that one of the main challenges in the simulation of this category of fluid is to control the numerical instabilities, during the convergence process. These instabilities are amplified when the elastic parameters are increased, mainly in geometries where the convective and extensional effects are important, like in an abrupt contraction (Dou and Phan-Thien, 2007).

Bromani et al. (2006) simulated a drilling fluid flow through a concentric annular, using the PTT Affine rheological model. Coradin (2007) also employed the PTT Affine model to simulate a drilling fluid flow through an abrupt contraction. In both cases, when the elastic parameters were increased, the convergence of the simulations became extremely difficult.

Through the last two decades, some techniques were developed to improve the numerical convergence during the simulation of viscoelastic fluid flows. Many works (Hua-Shu and Phan-Thien, 2001) showed that formulations like BSD (Both Sides Diffusion), presented by Phan-Thien et al. (2004), EVSS (Rajagopalan, 1990) and DEVSS (Guenette and Fortin, 1995), improve significantly the rate of convergence of the simulation of elastic fluids, by reducing spurious oscillations.
This paper presents the simulation and analysis of a laminar viscoelastic flow, evaluating the influence of the Deborah number on the velocity and stress fields, friction factor and entry length. To accomplish this purpose the numerical instabilities and the computational costs must be reduced by the application of BSD scheme.

The numerical simulations were performed using the commercial software PHOENICS-CFD. This software is based on the Finite Volume Method (Patankar,1980) and it has a partially opened code which allows users to modify or add routines.

2. MATHEMATICAL FORMULATION

2.1. Geometry

Fig. 1 shows the annular pipe geometry and the cylindrical coordinates system, employed in this problem. \( L \) is the total tube length and \( r_i \) and \( r_o \) are the inner and the outer radii, respectively.

![Figure 1. Annular pipe geometry and the cylindrical coordinates system.](image)

2.2. Governing equations

To solve the proposed problem the assumptions adopted are: steady, incompressible, laminar, axisymmetric, bidimensional and isothermal flow; symmetric stress tensor; shear stresses over \( \theta \) direction are disregarded and gravitational effects are neglected.

The continuity (1) and momentum (2) equations, in cylindrical coordinates, are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]  

(1)

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + (\nabla \cdot \rho \mathbf{v}) = \nabla \cdot (\mathbf{\sigma} - \rho \mathbf{I}) + \rho \mathbf{g}
\]  

(2)

where \( \mathbf{v} \) is the velocity vector, \( \rho \) is the fluid density, \( \mathbf{\sigma} \) is the extra stress tensor, \( \mathbf{g} \) is the gravity vector, \( \rho \) represents the thermodynamic pressure and \( \mathbf{I} \) is the identity matrix.

Applying the assumptions presented above, the continuity equation becomes:

\[
\frac{1}{r} \frac{\partial}{\partial r} (r V_r) + \frac{\partial}{\partial z} V_z = 0
\]  

(3)

where \( V_r \) and \( V_z \) are the radial and axial velocity components, respectively.

The scalar components of the momentum equation, over the radial and axial directions, become:

\[
\begin{align*}
\rho \left( V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z} \right) &= \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) + \frac{\partial}{\partial z} \tau_{rz} - \frac{\tau_{ro}}{r} \right] - \frac{\partial p}{\partial r} \quad (r) \\
\rho \left( V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} \right) &= \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{\partial}{\partial z} \tau_{zz} \right] - \frac{\partial p}{\partial z} \quad (z)
\end{align*}
\]  

(4)

(5)

where \( \tau_{rr} \), \( \tau_{rz} \) and \( \tau_{zz} \) are the normal stresses and \( \tau_{rz} \) is the shear stress.
The boundary conditions for the pipe flow are: a constant velocity profile at the inlet; a no-slip and impermeable conditions at the pipe wall and a null pressure at the pipe outlet.

2.3. PTT model equation

The PTT constitutive equation, proposed by Phan-Thien and Tanner (1977) can be expressed, for a viscoelastic material with a single relaxation time $\lambda$, like:

$$\tau + \xi (D\tau - \tau) + \frac{Y}{\lambda} \tau = 2GD$$

where $\tau$ is the Oldroyd’s upper convective derivative, $D$ is the deformation-rate tensor and $G$ the relaxation module, which is defined by the ratio between the zero-shear-rate apparent viscosity and the relaxation time.

In the present work, we adopt the simplified PTT model, known as PTT Affine (Tanner, 2002), which despises the slippage between the molecular network and the continuum medium ($\xi = 0$).

The function $Y$ depends on the rate of creation and destruction of the links between the polymeric chains. This function can assume a linear or an exponential form. In the present work the linear form will be used:

$$Y = \left(1 + \frac{\varepsilon}{G} \text{tr}(\tau)\right)$$

where $\varepsilon$ is the elongational parameter of the PTT model governing the extensional flow response.

Finally, the scalar components of the stress tensor become:

$$\rho r r : \quad V_r \frac{\partial \tau_{rr}}{\partial r} + V_z \frac{\partial \tau_{zz}}{\partial z} = 2\left(\tau_{rr} \frac{\partial V_r}{\partial r} + \tau_{zz} \frac{\partial V_z}{\partial z}\right) + \frac{Y}{\lambda} \tau_{rr} = 2G \frac{\partial V_r}{\partial r}$$

$$\rho \theta \theta : \quad V_r \frac{\partial \tau_{r\theta}}{\partial r} + V_z \frac{\partial \tau_{z\theta}}{\partial z} = \frac{\partial V_r}{\partial r} \frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial V_z}{\partial z} \tau_{r\theta} = 2G \frac{V_r}{r}$$

$$\rho z z : \quad V_r \frac{\partial \tau_{rz}}{\partial r} + V_z \frac{\partial \tau_{r\theta}}{\partial z} = 2\left(\tau_{rz} \frac{\partial V_r}{\partial r} + \tau_{r\theta} \frac{\partial V_z}{\partial z}\right) + \frac{Y}{\lambda} \tau_{rr} = 2G \frac{\partial V_r}{\partial r}$$

$$\rho r z : \quad V_r \frac{\partial \tau_{rz}}{\partial r} + V_z \frac{\partial \tau_{r\theta}}{\partial z} = \left(\tau_{rz} \frac{\partial V_r}{\partial r} + \tau_{r\theta} \frac{\partial V_z}{\partial z}\right) + \frac{Y}{\lambda} \tau_{rr} = G \left(\frac{\partial V_r}{\partial r} + \frac{\partial V_z}{\partial z}\right)$$

2.4. Both Sides Diffusion Scheme (BSD Scheme)

The Both Sides Diffusion Scheme (BSD) is discussed by Phan-Thien et al. (2004). In this scheme two equal diffusive terms are introduced in the momentum equation (2), which leads to the following expression:

$$\frac{\partial}{\partial t} \rho \mathbf{v} + (\nabla \cdot \rho \mathbf{v} \mathbf{v}) - \nabla \cdot (\eta \nabla \mathbf{v}) = -\nabla \cdot (\rho \mathbf{I}) + \nabla \cdot (\sigma - \eta \nabla \mathbf{v}) + \rho \mathbf{g}$$

BSD and other convergence schemes aim to improve the unstable numerical convergence, by amplifying the elliptic operator in the momentum equation. BSD scheme propitiates a severe reduction in the frequency and amplitude of the spurious oscillations, during the solution of the complex non-linear system. The non-linear system is the set of coupled equations (3) - (5) and (8) - (11).
According to Phan-Thien et al. (2004), if one is interested in the viscoelastic steady flow simulation, the BSD and EVSS (Rajagopalan, 1990) schemes are valid options and they improve significantly the numerical convergence. Nevertheless, Phan-Thien et al. (1998) state that BSD scheme for high stress gradients or more singular geometric configurations is unstable. Considering the proposed problem in this paper, the BSD scheme seems to be appropriated.

3. NUMERICAL METHOD

The expressions (3) - (5) and (8) - (11) are solved using the software PHOENICS-CFD and implemented in the following general form:

$$\frac{\partial}{\partial t} (\rho \phi) + \nabla \cdot (\rho \mathbf{v} \phi) = \nabla \cdot (\Gamma_\phi \nabla \phi) + S_\phi + P_\phi \tag{13}$$

In equation (13), $\Gamma_\phi$ is the diffusive coefficient, $S_\phi$ and $P_\phi$ are the source terms. The $\Gamma_\phi$, $S_\phi$ and $P_\phi$ terms assume different forms depending on the $\phi$ variable, which can be the velocity or stress components. Equation (13) first term represents the transient phenomenon, followed by the term related to the convective acceleration. In the right side appear the diffusive and source terms. The divergent operator ($\nabla \phi$) represents the net quantity of $\phi$ crossing the boundaries of the finite volume.

PHOENICS software allows users to activate and deactivate each terms presented in equation (13). Patankar (1980) states that all the terms, which can not be adjusted to the general differential equation, must be implemented as a part of the source term. Therefore, the governing equations terms which can not be adjusted to the diffusive or convective terms are introduced in the source term $S_\phi$ (Spalding, 1994).

After the discretization of governing equations by the Finite Volume Method (Patankar, 1980), the PHOENICS-CFD software employs the SIMPLEST solution algorithm to handle the pressure-velocity coupling. The momentum equation is successively corrected until the reduction of the residue below a fixed value (PHOENICS, 1991). The Hybrid convective scheme is employed to interpolate the scalar and vectorial variables. The coefficient matrix is solved by the TDMA line-by-line method.

The numerical validation is based on the analytical solution proposed by Pinho and Oliveira (2000).

4. RESULTS

4.1. Monitoring the convergence process

To monitor the convergence process, the evolution of the percentage relative error was analyzed during the iterative solution. The percentage relative error was calculated by the following expression

$$\text{Relative Error} (\%) = \left( \frac{\phi_{\text{new}} - \phi_{\text{old}}}{\phi_{\text{new}} + 10^{-4}} \right) \times 100 \leq 10^{-4} \tag{14}$$

The relative error of all finite volumes was summed for certain iteration. After that, the average relative error was calculated by the ratio between the previous sum and the total number of finite volumes.

this sum was divided by the total finite volumes number in the problem domain, giving the average relative error.

Error evolution was monitored for the PTT+BSD code and PTT code. The Reynolds number was fixed in 500, with a uniform velocity profile at the tube inlet. The mesh employed was $80 \times 80$ (80 volumes in the radial direction and 80 volumes in the axial direction).

The fluid elasticity was modified increasing the Deborah number. The Deborah number is defined by the ratio between the material characteristic relaxation time ($\lambda$) and the time period of the stress or deformation application.

Extensional parameter $\epsilon$ was fixed in 0.25. According to Pinho (2003), this value is typical of concentrated polymer solutions and represents the maximum extensional behavior for the PTT model.

Figures 2a e 2b show that the BSD scheme reduces drastically the numerical oscillations. Therefore, the sub-relaxations employed in the simulations with PTT+BSD are less restricted than the ones applied to the conventional PTT. The adopted pressure sub-relaxation was $10^{-2}$ for all cases. The other variables sub-relaxation was $10^{-3}$ with BSD and $10^{-4}$ with conventional PTT. These sub-relaxations are the limit to obtain numerical convergence, for a large Deborah number range, and they depend on the employed numerical algorithm.
The convergence was monitored for different Deborah numbers. The average error evolution, for $De = 1$ and $De = 100$, are presented in Figure 2a and 2b, respectively. The convergence improvement with BSD scheme is remarkable, for both Deborah numbers. The monotonic behavior of the curves PTT+BSD carries a reduction in the number of iterations necessary for convergence, situation observed in the range of Deborah numbers simulated, $0.1 < De < 150$.

In general flow of drilling fluid presents low values of Deborah numbers. Despite that, this study also investigates the effects of high Deborah numbers.

One can observe that, for a viscoelastic developing flow in an annular tube, the BSD scheme improves dramatically the numerical rate of convergence, reducing one order of magnitude the number of iterations to reach the steady state. Comparing the conventional PTT and the PTT+BSD simulations, the computational cost demanded to process a single iteration is almost the same for both cases. Thus, reducing the iteration number we observe a proportional reduction in the computational cost. When the Deborah number is increased, the numerical oscillations become more intense, augmenting the computational cost and often making the convergence impossible. So, the implementation of BSD scheme has been a fundamental requirement to overcome the spurious oscillations faced during the numerical convergence.

4.2 Influence of the Deborah number

Starting the investigation about the influence of the Deborah number, we initially tested the numerical mesh refining, for $De = 50$. This Deborah value was adopted because the analytic solution, proposed by Pinho and Oliveira (2000), for a steady state condition, shows that Deborah numbers above 50 do not affect significantly the velocity and stress profiles, neither the friction factor.

After the test, the mesh choose for the study of the influence of Deborah number was $200 \times 200$, non-uniform, with refined regions close to the tube walls and at the tube inlet. This refinement is necessary because of the intense velocity and stress gradients close to the walls and because of the flow development at the tube inlet.

Convergence was obtained for all Deborah numbers simulated. When the Deborah number is increased, the simulations become slower but, as we will see in the next results, convergence continues even for $De = 150$.  

![Figura 2 – Evolution of the percentage relative error during the monitoring of the axial velocity convergence, with (a) $De = 1$ and (b) $De = 100$.](image)
Figura 3 – Effect of Deborah number on the non-dimensional axial velocity profile, for a fully developed flow. 200×200 non-uniform mesh, \( \varepsilon = 0.25 \).

Figura 4 – Influence of the Deborah number on the (a) non-dimensional shear stress and (b) non-dimensional normal stress, for fully developed flow. 200×200 non-uniform mesh, \( \varepsilon = 0.25 \).

Fig. 3 depicts the influence of the Deborah number on the axial velocity profile. One observe that the higher \( De \), the flatter the profiles and the larger are the gradients near the walls.

In Fig. 4a and 4b one note the significant reduction in the shear and normal stresses, respectively, when Deborah number is increased. These profiles behavior is explained by the amplification of the shear-thinning effect. This effect implies in a viscosity reduction with the increasing deformation rate (Pinho and Oliveira, 2000).

In Fig 3 and 4, one can also note that the influence of the Deborah number is reduced between \( De = 20 \) and \( De = 50 \).

To study the influence of the Deborah number on the entrance length, the simulated range of Deborah was \( 0.1 < De < 150 \).

The elasticity (\( De \)) also influence the entrance length and the friction factor, as observed in figures 5a and 5b, respectively.
Figura 5 - (a) Influence of the Deborah number on the non-dimensional entrance length \( \left( \frac{Le}{D_h} \right) \) and (b) Fanning friction factor. 200×200 non-uniform mesh.

Fig. 5b shows a remarkable fall on the \( f \cdot Re \) between \( 0 < \sqrt{\varepsilon}De < 5 \), as a consequence of the increasing shear-thinning effect. The friction factor reduction can be very useful to the industry, reducing the energy losses by friction. The discrepancy, in figure 5b, between numerical and analytical solutions is about 0.15%.

The influence of the Deborah number on the entrance length is presented in Figure 5a. The analysis of this influence for viscoelastic fluids is rare in the literature and provides interesting information. Figure 5a shows the entrance length \( (Le) \) and the tube hydraulic diameter \( (D_h) \) ratio, as a function of the Deborah number (in this case \( De \) ranges from 0.1 to 150).

To Deborah numbers lower than 50, the relation \( \frac{Le}{D_h} \times De \) is almost linear, and in the case of \( De \) greater than 50, the influence of this parameter on the entrance length decreases progressively. The equation (15) express a relation between the non-dimensional entrance length and the Deborah number, for \( 1 \leq De \leq 150 \), \( Re = 500 \) and \( \varepsilon = 0.25 \).

\[
\frac{Le}{D_h} = c_1 De^3 - c_2 De^2 + c_3 De - c_4
\]  

\( c_1 = 0.0036 \), \( c_2 = 1.323 \), \( c_3 = 168.01 \), \( c_4 = 138.63 \), with \( R^2=0.999 \).

As mentioned earlier, the influence of Deborah number on the velocity profiles (figure 3), stress (figure 4) and on the friction factor (figure 5a) reduces progressively. It happens because, for Deborah numbers over 50, the elastic effects are already very pronounced and saturated, so increasing Deborah number is not more significant.

5. CONCLUSIONS

In this paper we presented a numerical study of a PTT viscoelastic fluid flow through an annular concentric pipe. The numerical simulations were developed with PHOENICS-CFD software. The objective of the present study is to evaluate the influence of the Deborah number on the flow pattern.

The problem of spurious oscillations during the numerical convergence for high Deborah number was surpassed using BSD scheme. The BSD scheme promotes a considerable reduction on the number of iterations necessary to reach the convergence. This reduction saves a proportional computational cost, because the simulation time for a single iteration, is similar for conventional PTT and PTT+BSD. Also, with BSD scheme was possible to simulate high values of Deborah number (even \( De = 150 \)).

For Deborah numbers over 50, the elastic effects are already saturated. Thus, increasing Deborah number is not more meaningful.

The friction factor falls quickly when \( De \) is smaller than 5. After \( De = 5 \), the reduction becomes much more slight.
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